CBSE Class 11 Mathematics

Important Questions

Chapter 7

Permutations and Combinations

1 Marks Questions

1. Evaluate $4! - 3!$

Ans. $4!-3!=4.3!-3!$

$=(4-1).3!$

$=3.3!=3\times3\times2\times1$

$=18$

2.If ${}^nc_a={}^nc_b$ **find** n

Ans. ${}^nC_a={}^nC_b\Rightarrow{}^nC_a={}^nC_{n-b}$

$a=n-b$

$n=a+b$

3.The value of 0! Is?

Ans. $0!=1$

4.Given 5 flags of different colours here many different signals can be generated if each signal requires the use of 2 flags. One below the other

Ans. First flag can be chosen is 5 ways

Second flag can be chosen is 4 ways

By $F.P.C.$ total number of ways $=5\times4=20$

5.How many 4 letter code can be formed using the first 10 letter of the English alphabet, if no letter can be repeated?

Ans. First letter can be used in 10 ways

Second letter can be used in 9 ways

Third letter can be used in 8 ways

Forth letter can be used in 7 ways

By $F.P.C.$ total no. of ways $=10.9.8.7$

$=5040$

6.A coin is tossed 3 times and the outcomes are recorded. How many possible out comes are there?

Ans. Total no. of possible out comes $=2\times 2\times 2=8$

7.Compute $\frac{8!}{6!\times 2!}$

Ans. $\frac{8!}{6!2!}=\frac{8.7.6!}{6!.2.1}$

$=4\times 7=28$

8.If $^nC_8={}^nC_2$. find $^nC_2$.

Ans. Given

$^nC_8={}^nC_2\Rightarrow {}^nC_{n-8}={}^nC_2$

$n-8=2$

$n=10$

$$\therefore {}^{n}C_2 = {}^{10}C_2 = \frac{\lfloor 10}{\lfloor 10-2 \lfloor 2}$$

$$= \frac{10.9.\lfloor 8}{\lfloor 8 \times 2.1} = 5 \times 9 = 45$$

9.In how many ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colours.

Ans. No. of ways of selecting 9 balls

$$= {}^{6}C_3 \times {}^{5}C_3 \times {}^{5}C_3$$

$$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$$

$$= \frac{6.5.4\lfloor 3}{6.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3}$$

$$= 20 \times 10 \times 10 = 2000$$

10.Find r, **if** $5.{}^{4}P_r = 6.{}^{5}P_{r-1}$

Ans. $5.\ {}^{4}p_r = 6.\ {}^{5}p_{r-1}$

$$\Rightarrow 5.\frac{\lfloor 4}{\lfloor 4-r} = 6.\frac{\lfloor 5}{\lfloor 5-r+1}$$

$$\Rightarrow \frac{5.\lfloor 4}{\lfloor (4-r)} = \frac{6.5.\lfloor 4}{\lfloor 6-r}$$

$$\Rightarrow \frac{1}{\lfloor 4-r} = \frac{6}{(6-r)(5-r)\lfloor 4-r}$$

$$\Rightarrow (6-r)(5-r) = 6$$

$\Rightarrow 30-6r-5r+r^2=6$

$\Rightarrow r^2-11r-5r+r^2=6$

$\Rightarrow r^2-8r-3r+24=0$

$\Rightarrow r(r-8)-3(r-8)=0$

$\Rightarrow (r-3)(r-8)=0$

$r=3$ or $r=8$

$\therefore$ r = 3

$r=8$ Rejected. Because if we put $r=8$ the no. in the factorial is –ve.

11.If $\frac{1}{6!}+\frac{1}{7!}+=\frac{x}{8!}$ **find** x

Ans. $\frac{1}{6!}+\frac{1}{7.6!}=\frac{x}{8.7.6!}$

$\frac{1}{6!}\left[1+\frac{1}{7}\right]=\frac{x}{56.6!}$

$\frac{8}{7}=\frac{x}{56}$

$x=8\times 8=64$

12.Write relation between ${}^{n}p_{r}$. **and** ${}^{n}C_{r}$

Ans. ${}^{n}P_{r}={}^{n}C_{r}\times \lfloor r$

13.What is $\lfloor n$

Ans. $\lfloor n$ Is multiplication of n consecutive natural number

$\lfloor n = n(n-1)(n-2)(n-3)......5.4.3.2.1$

14.If $^{n}C_{o} = 1$ **what is the value of** $^{99}C_{o}$

Ans. $^{n}C_{o} = 1$ then $^{99}C_{o} = 1$

15.How many words, with or with not meaning each of 2 vowels and 3 consonants can be flamed from the letter of the word DAUGHTER?

Ans. In the word DAUGHTER There are 3 vowels and 5 consonants out of 3 vowels, 2 vowels can be selected in $^{3}C_{2}$ ways and 3 consonants can be selected in $^{5}C_{3}$ ways 5 letters 2 vowel and 3 consonant can be arranged in 5! Ways

$\therefore$ Total no. of words $= {}^{3}C_{2} \times {}^{5}C_{3} \times 5!$

$$= \frac{\lfloor 3}{\lfloor 1 \lfloor 2} \times \frac{\lfloor 5}{\lfloor 2 \times \lfloor 3} \times \lfloor 5$$

$$= \frac{3.\lfloor 2}{\lfloor 2} \times \frac{5.4.\lfloor 3}{\lfloor 2.\lfloor 3} \times 5.4.3.2.1$$

$$= 3600$$

16.Convert the following products into factorials $5 \times 6 \times 7 \times 8 \times 9$

Ans. $\dfrac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9}{1 \times 2 \times 3 \times 4}$

$$= \frac{\lfloor 9}{\lfloor 4}$$

17.Evaluate $\dfrac{n!}{(n-r)!}$, when $n = 5,\ r = 2$

Ans. $\frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor 5}{\lfloor 5-2} = \frac{\lfloor 5}{\lfloor 3}$

$= \frac{5.4.\lfloor 3}{\lfloor 3} = 20$

18.Evaluate ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$

Ans. $\left({}^{15}C_8 + {}^{15}C_9\right) - \left({}^{15}C_6 + {}^{15}C_7\right) \quad \left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

$= {}^{16}C_9 - {}^{16}C_7 = 0 \qquad \left[\because {}^{16}C_9 = {}^{16}C_7\right]$

19.What is the value of ${}^{n}C_o + {}^{n}C_1 + {}^{n}C_2 + \ldots\ldots + {}^{n}C_n$

Ans. 2^n

20.Find n **if** ${}^{2n}C_3 : {}^{n}C_3 = 11:1$

Ans. Given ${}^{2n}C_3 : {}^{n}C_3 = 11:1 \Rightarrow \frac{\lfloor 2n}{\lfloor 3 \lfloor 2n-3} \times \frac{\lfloor 3 \lfloor n-3}{\lfloor n} = \frac{11}{1}$

$\Rightarrow \frac{2n(2n-1)(2n-2)\lfloor 2n-3}{\lfloor 2n-3} \times \frac{\lfloor n-3}{n(n-1)(n-2)\lfloor n-3} = \frac{11}{1}$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = \frac{11}{1} \Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1}$

$\Rightarrow 11n - 22 = 8n - 4 \Rightarrow 3n = 18$

$\Rightarrow n = 6$

21.Evaluate ${}^{10}C_7 + {}^{10}C_6$

Ans. ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

${}^{10}C_7 + {}^{10}C_6 = {}^{10+1}C_7$

$= {}^{11}C_7 = \frac{\underline{|11}}{\underline{|11-7}\underline{|7}}$

$= \frac{\underline{|11}}{\underline{|4}\ \underline{|7}} = \frac{11.10.9_3.8.7.\underline{|7}}{4.3.2.1.\underline{|7}} = 330$

22.If $1 \le r \le n$ then what is the value of $\frac{n}{r}\ {}^{n-1}C_{r-1}$

Ans. ${}^{n}C_r$

H	T	U
7	8	9

23.How many 3 digit numbers can be formed by using the digits 1 to 9 if no. digit is repeated

Ans.

$9 \times 8 \times 7 = 504$

24.Convert into factorial 2.4.6.8.10.12

Ans. $2\times1.\ 2\times2.\ 2\times3.\ 2\times4.\ 2\times5.\ 2\times6$

$= 2^6 [1.2.3.4.5.6] = 2^6\ \underline{|6}$

25.How many words with or without meaning can be formed using all the letters of the word 'EQUATION' at a time so that vowels and consonants occur together

Ans. In the word 'EQUATION' there are 5 vowels [A.E.I.O.U.] and 3 consonants [Q.T.N]

Total no. of letters = 8

Arrangement of 5 vowels = $\lfloor 5$

Arrangements of 3 consonants = $\lfloor 3$

Arrangements of vowels and consonants = $\lfloor 2$

$\therefore$ Total number of words $= \lfloor 5 \times \lfloor 3 \times \lfloor 2$

$= 5.4.3.2.1 \times 3.2.1 \times 2.1 = 1440$

CBSE Class 12 Mathematics

Important Questions

Chapter 7

Permutations and Combinations

4 Marks Questions

1. How many words, with or without meaning can be made from the letters of the word MONDAY. Assuming that no. letter is repeated, it

(i) 4 letters are used at a time

(ii) All letters are used but first letter is a vowel?

Ans. Part-I In the word MONDAY there are 6 letters

$\therefore n = 6$

4 letters are used at a time

$\therefore r = 4$

Total number of words $= {}^nP_r$

$$= {}^6P_4 = \frac{\lfloor 6}{\lfloor 6-4}$$

$$= \frac{\lfloor 6}{\lfloor 2} = \frac{6.5.4.3.\lfloor 2}{\lfloor 2} = 360$$

Part-II All letters are used at a time but first letter is a vowel then OAMNDY

2 vowels can be arranged in 2! Ways

4 consonants can be arranged in 4! Ways

$\therefore$ Total number of words $= 2! \times 4!$

$= 2 \times 4.3.2.1 = 48$

2.Prove that $^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}$

Ans. Proof L.H.S.

$$^{n}C_{r}+{}^{n}C_{r-1}=\frac{\lfloor n}{\lfloor n-r\lfloor r}+\frac{\lfloor n}{\lfloor n-r+1\lfloor r-1}$$

$$=\frac{\lfloor n}{\lfloor(n-r)\ r\lfloor r-1}+\frac{\lfloor n}{(n-r+1)\lfloor n-r\lfloor r-1}$$

$$=\frac{\lfloor n}{\lfloor n-r\lfloor r-1}\qquad\left[\frac{1}{r}+\frac{1}{n-r+1}\right]$$

$$=\frac{\lfloor n}{\lfloor n-r\lfloor r-1}\qquad\left[\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right]$$

$$=\frac{\lfloor n(n+1)}{\lfloor n-r\,(n-r+1)\lfloor r-1\,r}$$

$$=\frac{\lfloor n+1}{\lfloor n+1-r\lfloor n-r}={}^{n+1}C_{r}$$

3. A bag contains 5 black and 6 red balls determine the number of ways in which 2 black and 3 red balls can be selected.

Ans. No. of black balls =5

No. of red balls = 6

No. of selecting black balls = 2

No. of selecting red balls = 3

Total no. of selection$= {}^{5}C_{2}\times{}^{6}C_{3}$

$$= \frac{\underline{|5}}{\underline{|5-2}\underline{|2}} \times \frac{\underline{|6}}{\underline{|6-3}\ \underline{|3}}$$

$$\frac{5\times 4\times 3!}{3!\times 2}\times\frac{6\times 5\times 4\times 3!}{3!\times 3\times 2}=200$$

4.In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Ans. Let us first seat 0 the 5 girls. This can be done in 5! Ways

X G X G X G X G X G X

There are 6 cross marked placer and the three boys can be seated in ${}^6P_3$ ways

Hence by multiplication principle

The total number of ways

$$= 5! \times {}^6P_3 = 5! \times \frac{6!}{3!}$$

$$= 4\times 5\times 2\times 3\times 4\times 5\times 6$$

$$= 14400$$

5. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE

Ans. In the INVOLUTE there are 4 vowels, namely I.O.E.U and 4 consonants namely M.V.L and T

The number of ways of selecting 3 vowels

Out of $4 = {}^4C_3 = 4$

The number of ways of selecting 2 consonants

Out of $4 = {}^4C_2 = 6$

$\therefore$ No of combinations of 3 vowels and 2 consonants $= 4 \times 6 = 24$

5 letters 2 vowel and 3 consonants can be arranged in 5! Ways

Therefore required no. of different words $= 24 \times 5! = 2880$

6. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements

(i) do the words start with P

(ii) do all the vowels always occur together

Ans. The number of letters in the word INDE PENDENCE are 12 In which E repeated in 4 times. N repeated 3 times. D repeated 2 times

(i) If the word starts with P The position of P is fined

Then the no. of arrangements $= \frac{11!}{4!3!2!} = 138600$

(ii) All the vowels always occur together There are 5 vowels in which 4 E's and 1 I

EEEEI NDPNDNC

$\therefore$ Total letters are 8 letters Can be arranged in $= \frac{8!}{3!2!}$ ways

Also 5 vowels can be arranged in $= \frac{5!}{4!}$ ways.

$\therefore$ required number of arrangements

$$= \frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$$

7.Find n if ${}^{n-1}P_3 : {}^{n}P_4 = 1:9$

Ans. Given

$^{n-1}P_3 : {}^nP_4 = 1:9$

$\frac{\lfloor n-1}{\lfloor n-1-3} : \frac{\lfloor n}{\lfloor n-4} = 1:9$

$\frac{\frac{\lfloor n-1}{\cancel{\lfloor n-4}}}{\frac{\lfloor n}{\cancel{\lfloor n-4}}} = \frac{1}{9}$

$\frac{\cancel{\lfloor n-1}}{n\cancel{\lfloor n-1}} = \frac{1}{9}$

$n = 9$ s

8. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans. Given total no. of players =17

5 players can bowl.

$\therefore$ no. of bat's man =17-5=12

Out of 5 bowlers 4 can be choose in $^5C_4$ ways

Out of 12 bat's man (11-4)=7 bat's man can be choose in $^{12}C_7$ ways

Total no. of selection of 11 players

$= {}^5C_4 \times {}^{12}C_7$

$= \frac{\lfloor 5}{\lfloor 5-4 \lfloor 4} \times \frac{\lfloor 12}{\lfloor 12-7 \lfloor 7}.$

$$5\times\frac{12.11.10.9.8.\underline{|7}}{5.4.3.2.1\underline{|7}}=3960$$

9. How many numbers greater than 1000000 can be formed by using the digits 1,2,0,2,4,2,4?

Ans. Given digits $1, 2, 0, 2, 4, 2, 4$ Are 7

The no. of arrangements of 7 digits $=\frac{7!}{3!21.}=420$

If 0 is in extreme left position.

The no. of arrangements of 6 digits $=\frac{6!}{3!2!}=60$

$\therefore$Required numbers $=420-60=360$

10. In how many ways can the letters of the word ASSA SS IN ATION be arranged so that all the S's are together?

Ans. In the word ASSA SSI N ATION there are 13 letters

But all S are together. Then no. of letters 10 [4 S take 1]then required no. of arrangements

$$=\frac{10!}{3!2!2!} \quad =151200$$

11. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be flamed from the alphabet?

Ans. No. of vowels = 5

No. of consonants = 21

2 vowels can be selected in ${}^5C_2$ ways

2consonents can be selected in ${}^{21}C_2$ ways

No. of arrangements of 4 letters [2 vowel and 2 consonants] =4!

$\therefore$ Required no. of wards $= {}^{5}C_2 \times {}^{21}C_2 \times 4!$

$$= \frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \frac{\lfloor 21}{\lfloor 19 \lfloor 2} \times 4.\lfloor 3$$

$$= \frac{5.4.3.\lfloor 2 \times 21.20.\lfloor 19 \times 4.\lfloor 3}{\lfloor 3.\lfloor 2 \quad \lfloor 19 \times 2}$$

$$= 50400$$

12. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Ans. In the word MISSISSIPPI no. of letters = 11

In which 4 I's, 4 S's and 2P

$\therefore$ Total no. of words $= \frac{11!}{4!4!2!} = 34650$

When four I's come together

Then four I's as one letter

And other letters are 7

Then no. of words when four I's come together

$$= \frac{8!}{4!2!} = 840$$

Then the no. of permutations when four I's do not come together

$$= 34650 - 840 = 33810$$

13. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colors are in distinguishable?

Ans. Total no. of discs $= 4+3+2=9$

Out of 9 discs 4 are of some kind, 3are of same kind, 2 are of same kind

There for the number of arrangements $= \frac{9!}{4!3!2!} = 1260$

14. Find the number of permutations of the letters of the word ALLAHABAD.

Ans. In the word ALLAHABAD no. of letters = 9

In which four A's and two L's

$\therefore$ The no. of permutations $= \frac{9!}{4!2!} = 7560$

15. How many 4 letter code can be formed using the first 10 letters of the English alphabet if no letter can be repeated?

Ans. There are 10 letters of English alphabet

For making 4 letter code

First letter can be choose in 10 ways

Second letter can be choose in 9 ways

Third letter can be choose in 8 ways

Forth letter can be choose in 7 ways

By fundamental principle of multiplication

Total no. of code $= 10\times9\times8\times7 = 5040$

16. Determine the number of ways of choosing 5 cards out of a deck of 52 cards which

include exactly one ace.

Ans. In a deck of 52 cards, there are 4 aces and 48 other cards. Here, we have to choose exactly one ace 4 other cards.

The number of ways of choosing one ace out of 4 aces $= {}^4C_1$.

The number of ways of choosing 4 cards out of the other 48 cards $= {}^{48}C_4$.

Corresponding to one way of choosing an ace, there are ${}^{48}C_4$ ways of choosing 4 other cards. But there are ${}^4C_1$ ways of choosing aces, there fore, the required number of ways

$$= {}^4C_1 \times {}^{48}C_4 = \frac{4}{1} \times \frac{48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4} = 778320$$

17.How many numbers greater than 56000 and farmed by using the digits 4,5,6,7,8, no digit being repeated in any number?

Ans. Number greater than 56000 and farmed by using the digits 4,5,6,7,8 are of types

$5(6/7/8)\times\times\times$ or $(6/7/8)\times\times\times$

Now numbers of type $5(6/7/8)\times\times\times$ are $1\times3\times3\times2\times1 = 18$ in number

Number of type $(6/7/8)\times\times\times\times$ are $3\times4\times3\times2\times1 = 72$ in number

Hence required number of numbers is $18 + 72 = 90$

18.Find n, if $\frac{\lfloor n}{\lfloor 2 \lfloor n-2}$ and $\frac{\lfloor n}{\lfloor 4 \lfloor n-4}$ are in the ratio 2:1

Ans. Given $\frac{\lfloor n}{\lfloor 2 \lfloor n-2} : \frac{\lfloor n}{\lfloor 4 \lfloor n-4} = 2:1$

$$\Rightarrow \frac{\lfloor n}{\lfloor 2 \lfloor n-2} \times \frac{\lfloor 4 \lfloor n-4}{\lfloor n} = \frac{2}{1}$$

$$\Rightarrow \frac{4\times3\times\lfloor 2\times\lfloor n-4}{\lfloor 2\times(n-2)\times(n-3)\times\lfloor n-4}=\frac{2}{1}$$

$$\Rightarrow \frac{4\times3}{(n-2)(n-3)}=\frac{2}{1}\Rightarrow(n-2)(n-3)=6$$

$$\Rightarrow n^2-5n=0\Rightarrow n(n-5)=0$$

$$\Rightarrow n=0 \text{ or } n=5$$

But, for $n=0$, $\lfloor n-2$ and $\lfloor n-4$ are not meaningful, therefore, $n=5$.

19. Prove that $\lfloor 2n=1.3.5......(2n-1).2^n.\lfloor n$

Ans. $\lfloor 2n=1.2.3.4.5.6......(2n-1)(2n)$

$$=[1.3.5......(2n-1)][2.4.6......2n]$$

$$=[1.3.5.......(2n-1)][(2.1)(2.2)(2.3)......(2n)]$$

$$=1.3.5.......(2n-1).2^n.(1.2.3......n)$$

$$=1.3.5.......(2n-1).2^n.\lfloor n, \text{ as desired.}$$

20. How many 4 letter words with or without meaning, can be formed out of the letters of the word 'LOGARITHMS', if repetition of letters is not allowed?

Ans. There are 10 letters in the word 'LOGARITHMS'

For making 4 letter word we take 4 at a time

$\therefore$ No. of arrangements 10 letters taken 4 at a time

$$={}^{10}P_4=5040$$

21. From a class of 25 students 10 are to be chosen for an excursion Party. There are 3

students who decide that either all of them will join or none of them will join. In how many ways can excursion party be chosen?

Ans. Total no. of students = 25

No. of students to be selected = 10

I case :

3 students all of them will join the excursion party.

Then remaining 7 students will be selected out of (25-3 = 22) in ${}^{22}C_7$ ways

II case :

All 3 students will not join the party then 10 students will be selected in ${}^{22}C_{10}$ ways

Total no. of selection $= {}^{22}C_7 + {}^{22}C_{10}$

$$= \frac{\lfloor 22}{\lfloor 15 \lfloor 7} + \frac{\lfloor 22}{\lfloor 12 \lfloor 10} = 817190$$

22. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Ans. No. of red balls = 6

No. of white balls = 5

No. of blue balls = 5

No. of selecting each colour balls = 3

$\therefore$ required no. of selection = ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$$

$$=\frac{6.5.4.\lfloor 3}{\lfloor 3.3.2.1}\times\frac{5.4^{2}\lfloor 3}{2.1.\lfloor 3}\times\frac{5.^{2}4.\lfloor 3}{2.1.\lfloor 3}$$

$$=5\times4\times5\times2\times5\times2$$

$$=20\times10\times10=2000$$

23. Find the number of 3 digit even number that can be made using the digits 1, 2, 3, 4, 5, 6, 7, if no digit is repeated?

Ans. For making 3 digit even numbers unit place of digit can be filled in 3 ways. Ten's place of digit can be filled in 5 ways. Hundred place of digit can be filled in 4 ways. $\therefore$ Required number of 3 digit even number $=3\times5\times4\ =60$

24. If ${}^{n}P_{r}={}^{n}P_{r+1}$ and ${}^{n}C_{r}={}^{n}C_{r-1}$ **find the values of** n and r

Ans. Given that ${}^{n}P_{r}={}^{n}P_{r+1}$

$$\Rightarrow\frac{\lfloor n}{\lfloor n-r}=\frac{\lfloor n}{\lfloor n-r-1}$$

$$\Rightarrow\frac{1}{(n-r)\ \lfloor n-r-1}=\frac{1}{\lfloor n-r-1}$$

$$=n-r=1......(i)$$

And ${}^{n}C_{r}={}^{n}C_{r-1}$

$$\Rightarrow\frac{\lfloor n}{\lfloor n-r\ \lfloor r}=\frac{\lfloor n}{\lfloor n-r+1\ \lfloor r-1}$$

$$\Rightarrow\frac{1}{r}=\frac{1}{n-r+1}$$

$$\Rightarrow n-r+1=r\Rightarrow n-2r=-1......(ii)$$

Solving eq. (i) and (ii)

$n=3, \quad r=2$

25. Prove that the product r of consecutive positive integer is divisible by $\lfloor r$

Ans. Suppose & consecutive positive integers are $(n+1),\ (n+2),\ \ldots\ldots(n+r)$

Then product $=(n+1).(n+2).(n+3)\ldots\ldots(n+r)$

$$=\frac{\lfloor n\ \ (n+1).(n+2).(n+3)\ldots\ldots(n+r)}{\lfloor n}$$

$$=\frac{1.2.3\ldots\ldots n.(n+1)(n+2)(n+3)\ldots\ldots(n+r)}{\lfloor n}$$

$$=\frac{\lfloor n+r}{\lfloor n}=\frac{\lfloor n+r}{\lfloor r\ \lfloor n+r-r}\quad \lfloor r$$

$=\left({}^{n+r}C_r\right)\ \lfloor r$ which is divisible by $\lfloor r$

CBSE Class 12 Mathematics

Important Questions

Chapter 7

Permutations and Combinations

6 Marks Questions

1. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has :

(i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?

Ans. Number of girls = 4

Number of boys = 7

Number of selection of members = 5

(i) If team has no girl

We select 5 boys

∴ Number of selection of 5 members

$$= {}^7C_5 = \frac{\lfloor 7}{\lfloor 5 \lfloor 2} = 21$$

(ii) At least one boy and one girl the team consist of

Boy	Girls
1	4
2	3
3	2
4	1

The required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140$

$= 441$

(iii) At least 3 girls

Girls	**Boys**
3	2
4	1

The required number of ways

$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7$

$= 91$

2. Find the number of words with or without meaning which can be made using all the letters of the word. AGAIN. If these words are writer as in a dictionary, what will be the 50th word?

Ans. In the word 'AGAIN' there are 5 letters in which 2 letters (A) are repeated

Therefore total no. of words $\frac{5!}{2!} = 60$

If these words are written as in a dictionary the number of words starting with Letter A. [A A G I N] $= 4! = 24$

The no. of wards starting with G [G A A I N] $= \frac{4!}{2!} = 12$

The no. of words starting with I [I A A G N] $= \frac{4!}{2!} = 12$

Now

Total words $= 24 + 12 + 12 = 48$

49^{th} Words = N A A G I

50^{th} Words = N A A I G

3. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of there

(i) Four cards one of the same suit

(ii) Four cards belong to four different suits

(iii) Are face cards.

(iv) Two are red cards & two are black cards.

(v) Cards are of the same colour?

Ans. The no. of ways of choosing 4 cards form 52 playing cards.

$$^{52}C_4 = \frac{52!}{4!48!} = 270725$$

(i) If 4 cards are of the same suit there are 4type of suits. [diamond club, spade and heart] 4 cards of each suit can be selected in $^{13}C_4$ ways

$\therefore$ Required no. of selection $= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4 = 2860$

(ii) If 4 cards belong to four different suits then each suit can be selected in $^{13}C_1$ ways

required no. of selection $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(iii) If all 4 cards are face cards. Out of 12 face cards 4 cards can be selected in $^{12}C_4$ ways.

$\therefore$ required no. of selection $^{12}C_4 = \frac{\lfloor 12}{\lfloor 8 \lfloor 4} = 495$

(iv) If 2 cards are red and 2 are black then. Out of 26 red card 2 cards can be selected in $^{26}C_2$

ways similarly 2 black card can be selected in ${}^{26}C_2$ ways

$\therefore$ required no. of selection $= {}^{26}C_2 \times {}^{26}C_2$

$$= \frac{26!}{2!4!} \times \frac{26!}{2!4!} = (325)^2$$

$$= 105625$$

(v) If 4 cards are of the same colour each colour can be selected in ${}^{26}C_4$ ways

Then required no. of selection

$$= {}^{26}C_4 + {}^{26}C_4 = 2 \times \frac{26!}{4!22!}$$

$= 29900$ s

4. If ${}^{n}P_r = {}^{n}P_{r+1}$ and ${}^{n}c_r = {}^{n}c_{r-1}$ find the value of n and r

Ans. Given that

$^nP_r = {}^nP_{r+1}$

$\Rightarrow \frac{\underline{|n}}{\underline{|n-r}} = \frac{\underline{|n}}{\underline{|n-r-1}}$

$\Rightarrow \frac{1}{(n-r)\underline{|n-r-1}} = \frac{1}{\underline{|n-r-1}}$

$\Rightarrow n-r=1..........(i)$

also $^nC_r = {}^nC_{r-1}$

$\Rightarrow \frac{\underline{|n}}{\underline{|n-r}\underline{|r}} = \frac{\underline{|n}}{\underline{|n-r+1}\underline{|r-1}}$

$\Rightarrow \frac{1}{\underline{|n-r}\ r\underline{|r-1}} = \frac{1}{(n-r+1)\underline{|n-r}\underline{|r-1}}$

$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$

$\Rightarrow n-2r=-1.........(ii)$

Solving eq (i) and eq (ii) we get $n=3$ and $r=2$

5. Find the value of n such that

$(i)\ {}^nP_5 = 42\,{}^nP_3,\ n>4$ $(ii)\ \frac{{}^nP_4}{{}^{n-1}P_4} = \frac{5}{3},\ n>4$

Ans (i) $^nP_5 = 42\,{}^nP_3$

$\Rightarrow \frac{\underline{|n}}{\underline{|n-5}} = 42\frac{\underline{|n}}{\underline{|n-3}}$

$\Rightarrow \frac{1}{\underline{|n-5}} = \frac{42}{(n-3)(n-4)\underline{|n-5}}$

$\Rightarrow (n-3)(n-4)=42$

$\Rightarrow n^2-4n-3n+12=42$

$\Rightarrow n^2 - 7n - 30 = 0$

$\Rightarrow n^2 - 10n + 3n - 30 = 0$

$\Rightarrow n(n-10) + 3(n-10) = 0$

$\Rightarrow (n+3)(n-10) = 0$

$n = -3$ or $n = 10$

$n = -3$ Is rejected

Because negative factorial is not defined $\therefore n = 10$

(ii)

$\frac{{}^{n}P_4}{{}^{n-1}P_4} = \frac{5}{3} \qquad n > 4$

$$\Rightarrow \frac{\frac{\underline{|n}}{\underline{|n-4}}}{\frac{\underline{|n-1}}{\underline{|n-5}}} = \frac{5}{3}$$

$$\Rightarrow \frac{\underline{|n}}{\underline{|n-4}} \times \frac{\underline{|n-5}}{\underline{|n-1}} = \frac{5}{3}$$

$$\Rightarrow \frac{n\,\underline{|n-1}}{(n-4)\,\underline{|n-5}} \times \frac{\underline{|n-5}}{\underline{|n-1}} = \frac{5}{3}$$

$\Rightarrow 3n = 5n - 20$

$\Rightarrow -2n = -20 \qquad \Rightarrow n = 10$

6. A committee of 7 has to be farmed from 9 boys and 4 girls in how many ways can this be done when the committee consists of

(i) Exactly 3 girls?

(ii) Attest 3 girls?

(iii) Atmost 3 girls?

Ans. No. of boys = 9

No. of girls = 4

But committee has 7 members

(i) When committee consists of exactly 3 girls

	Boys	**Girls**	
	9	4	
Selecting member	4	3	=7

$\therefore$ Required no. of selection $= {}^9C_4 \times {}^4C_3 = 504$

(ii) Attest 3 girls

	Boys	**Girls**	
	9	4	
Selecting member	4	3	=7
	3	4	=7

The required no. of selections $= {}^9C_4 \times {}^4C_3 + {}^9C_3 \times {}^4C_4$

$= 504 + 84$

$= 588$

(iii) Atmost 3 girls

	Boys	**Girls**	
	9	4	
	7	0	=7
Selecting member	6	1	=7
	5	2	=7

	4	3	=7

Then required no. of selection

$$= {}^9C_7 \times {}^4C_0 + {}^9C_6 \times {}^4C_1 + {}^9C_5 \times {}^4C_2 + {}^9C_4 \times {}^4C_3$$

$$= 36 + 336 + 756 + 504$$

$$= 1632$$

7. In how many ways can final eleven be selected from 15 cricket players' if

(i) there is no restriction

(ii) one of then must be included

(iii) one of them, who is in bad form, must always be excluded

(iv) Two of then being leg spinners, one and only one leg spinner must be included?

Ans. (i) 11 players can be selected out of 15 in ${}^{15}C_{11}$ ways

$= {}^{15}C_4$ ways $= \dfrac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4}$ ways=1365 ways

(ii) Since a particular player must be included, we have to select 10 more out of remaining 14 players.

This can be done in ${}^{14}C_{10}$ ways ${}^{14}C_4$ ways

$= \dfrac{14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4}$ ways = 1001 ways

(iii) Since a particular player must be always excluded, we have to choose 11 ways out of remaining 14

This can be done in ${}^{14}C_{11}$ ways $= {}^{14}C_3$ ways

$= \frac{14 \times 13 \times 12 \times 11}{1 \times 2 \times 3}$ ways $= 364$ ways.

(iv) One leg spinner can be chosen out of 2 in $^2C_1$ ways. Then we have to select 10 more players get of 13 (because second leg spinner can't be included). This can be done in $^{13}C_{10}$ ways of choosing 10 players. But these are $^2C_1$ ways of choosing a leg spinner, there fore, by multiplication principle of counting the required number of ways

$= {}^2C_1 \times {}^{13}C_{10}$

$= {}^2C_1 \times {}^{13}C_3 = \frac{2}{1} \times \frac{13 \times 12 \times 11}{1 \times 2 \times 3} = 572$

8. How many four letter words can be formed using the letters of the letters of the word 'FAILURE' so that

(i) F is included in each word

(ii) F is excluded in each word.

Ans. There are 7 letters in the word 'FAILURE'

(i) F is included for making each word using 4 letters

$\therefore$ F is already selected

Then other 3 letters can be selected out of 6 are $^6C_3$ ways

Also arrangements of 4 letters are 4! Ways so:

$\therefore$ Total no. of words $= {}^6C_3 \times 4! \;= 480$

(ii) F is excluded in each word

$\therefore$ Out of 6 letters are choose 4 letters in $^6C_4$ ways

Also arrangement of 4 letters are 4! Ways so:

$\therefore$ Total no. of words $= {}^6C_4 \times 4! = 360$

9. A committee of 5 is to be formed out of 6 gents and 4 Ladies. In how many ways this can be done, when

(i) at least two ladies are included?

(ii) at most two ladies are included?

Ans. No of person to form committee = 5

No. of gents = 6 and No. of ladies = 4

(i) At least two ladies are included

Ladies [4] Gents [6]

Either we Select $\rightarrow$ 2 and 3

or

$\rightarrow$ 3 and 2

Or

$\rightarrow$ 4 and 1

$\therefore$ required number of selection

$= {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$

$120 + 60 + 6 = 186$

(ii) At most two ladies are included?

Ladies [4] Gents [6]

Either we select 0 and 5

Or 1 and 4

Or 2 and 3

$\therefore$ required no. of selection.

$$= {}^4C_0 \times {}^6C_5 \times {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3$$

$$6+60+120=186$$

10. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) words start with P and with S

(ii) vowels are all together

(iii) There are always 4 letters between P and S?

Ans. In the word PERMUTATIONS there are 12 letters

(i) If the word start with P and end with S then position of P and S will be fixed. Then other 10 letters can be arranged in $\lfloor\underline{10}$ ways. But T occurs twice.

$\therefore$ no. of arrangements = $\dfrac{\lfloor\underline{10}}{\lfloor\underline{2}}$

=1814400 ways

(ii) Vowels are together?

No. of vowels in the word PERMUTATIONS are 5 which are [A, E, I, O, U]

$\therefore$ Vowels can be arranged is $\lfloor\underline{5}$ ways other letters are consonants out of 8 consonants 2 are repeated

$\therefore$ No. of arrangements of consonants $= \dfrac{\lfloor\underline{8}}{\lfloor\underline{2}}$

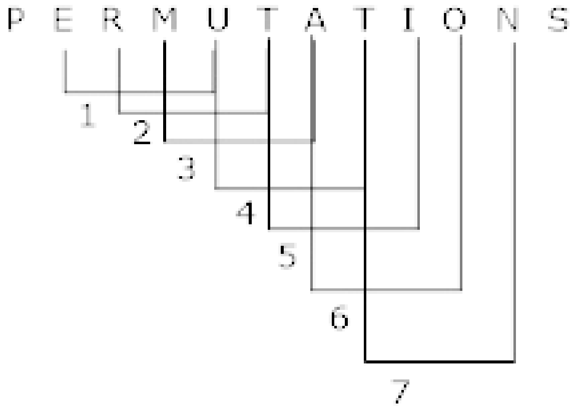
$\therefore$ requires no. arrangements $= \dfrac{\lfloor\underline{8}}{\lfloor\underline{2}} \times \lfloor\underline{5} = 2419200$

(ii) There are always 4 letters between P and S. in the word ‘PERMUTATIONS’

If 4 letters between P and S

Then P and S can be arranged in 2 ways other 10 letters can be arranged in $\frac{|\underline{10}}{|\underline{2}}$ ways

There are 7 pair 4 letters in the words PERMUTATIONS between P and S



$\therefore$ Required no. of arrangements = $\frac{|\underline{10}}{\cancel{|\underline{2}}} \times \cancel{2} \times 7$

= 2 5401600